Reduction of Spurious Responses of a Strip Resonator with Tilted Edges

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Abstract— In 1970, Mindlin obtained an exact solution of thickness twist modes in an infinite strip made of rotated Y cut of quartz, neglecting piezoelectric effects, by tilting side surfaces parallel to the X axis slightly off normal to the major planes. This angle is called Mindlin's angle. It means that excellent resonator characteristics of an infinite plate can be retained in a miniature strip resonator without excess spurious responses. Based on this concept, billions of miniature AT strip resonators have been produced. Similar designs have been extended to resonators of such crystals as Gallium Orthophosphate ($GaPO_4$) and Langasite family (LGS, LGN and LGT) in crystal class 32.

In Mindlin's exact solution, major spurious responses are due to face shear modes. Electrodes on major surfaces still piezoelectrically excite them, although their elastic coupling with thickness shear modes is null. Their distribution of piezoelectrically induced charge along the Z axis constitutes a harmonic cosine series.

Hence suitable choice of the width and the shape of electrodes can eliminate piezoelectric excitation of face shear modes. In particular, if the area of electrodes along the X axis is chosen to be proportional to a cosine distribution of a certain mode, responses of all other modes can be eliminated. Vormer applied a similar design to an X cut quartz resonator vibrating in length-extensional modes.

For a high frequency resonator, which has numerous spurious responses due to other modes, it is more practical to reduce not all but a few nearby responses. Adjusting of the width of a rectangular electrodes or a combination of several rectangular electrodes, which are easier to fabricate than a cosine electrode, can do this.

In a real resonator with a finite length along the X axis, there exist spurious responses due to thickness shear modes propagating along the X axis. No exact solution has been available. Approximate solutions and direct experimental observation of distribution of induced charge, however, suggested that the distribution of induced charge along the X axis of these spurious responses also approximately constituted a harmonic cosine series. Hence a similar design of electrodes can be applied to reduce responses of these modes.

I. INTRODUCTION

A wide plate of rotated-Y-cut made of crystal quartz has good features for a piezoelectric resonator. Only pure shear modes with the displacement parallel to the X axis are excited by electrodes on major surfaces with little disturbance due to spurious modes. By appropriate choice of a cut angle, such desirable feature as zero temperature coefficient of frequency can be obtained. AT and BT of quartz have been used for many decades.

There has been a strong demand for miniaturization of resonators. If a material is isotropic, side surfaces of normal to the plate and parallel to the X axis are traction free and hence can be cut out to make a rectangular strip resonator without disturbing the vibration having all the good features. This is not the case of anisotropic crystals and such a cut causes severe spurious responses.

In 1970, Mindlin found an exact solution of rotated-Y-cut of quartz strip by tilting side surfaces, of which angle is called Mindlin's angle, so that the traction free condition can be met. [1] Mindlin's solution is for pure elastic case without piezoelectricity and Mindlin's angle varies with temperature. Hence optimum combinations of the width vs thickness ratio, cut angle, tilt angle, and the length vs width ratio were to be pursued before an adoption of tilted side surface for practical use. Based on this design, billions of quartz strip resonators

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have been made in the last 30years. [2] Similar designs have been extended to resonators of such crystals as Gallium Orthophosphate ($GaPO_4$) and Langasite family (LGS, LGN and LGT) in crystal class 32. [3]

In Mindlin's exact solution, major spurious responses are due to face shear modes. Electrodes on major surfaces still piezoelectrically excite them, although their elastic coupling with thickness twist modes are null. Their distribution of piezoelectrically induced charge along the Z axis constitutes a harmonic cosine series.

Hence suitable choice of the width and the shape of electrodes can eliminate piezoelectric excitation of face shear modes. In particular, if the area of electrodes along the X axis is chosen to be proportional to a cosine distribution of a certain mode, responses of all other modes can be eliminated due to the orthogonality of harmonic cosine series. Vormer applied a similar design to an X cut quartz resonator vibrating in length-extensional modes at a low frequency. [4]

For a high frequency resonator, which has numerous spurious responses due to other modes, it is more practical to reduce not all but a few nearby responses. Adjusting the width of a rectangular electrode or a combination of several rectangular electrodes, which is easier to fabricate and register than a cosine electrode, can do this.

In a real resonator with a finite length along the X axis, there exist spurious responses due to thickness shear modes propagating along the X axis. Although no exact solution has been available, approximate solutions [5] [6] and a direct experimental observation of distribution of induced charge [7] suggested that the distribution of induced charge along the X axis of these spurious responses also approximately constituted a harmonic cosine series. Hence a similar design of electrodes can be applied to reduce responses of these modes.

After a brief introduction of Mindlin's exact solution, designs of electrodes to suppress face shear modes are discussed. Experimental results show a good agreement with the theory.

II. MINDLIN'S EXACT SOLUTION

The stress-strain relations for the rotated Y cut of crystals in class 32 including quartz are governed in the form of monoclinic symmetry

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{12} & c_{22} & c_{23} & c_{24} & 0 & 0 \\ c_{13} & c_{23} & c_{33} & c_{34} & 0 & 0 \\ c_{14} & c_{24} & c_{34} & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & c_{56} \\ 0 & 0 & 0 & 0 & c_{56} & c_{66} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} \tag{1}$$

where T is stress, S is strain and c is stiffness. In the thickness twist mode of a plate shown in Fig. 1, only the displacement u along the X axis is non-zero, other components of displacement, v and w, are zero and differentiation along the X axis is zero.

$$u \neq 0, \quad v = w = \frac{\partial}{\partial x} = 0 \tag{2}$$

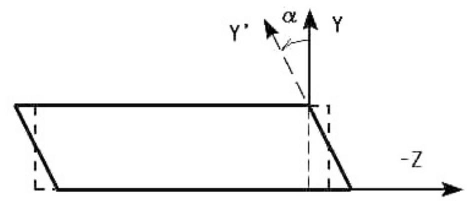

Fig. 1. Cross section of a strip

Then the stress-strain relations reduce to

$$\begin{pmatrix} T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} c_{55} & c_{56} \\ c_{56} & c_{66} \end{pmatrix} \begin{pmatrix} S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} c_{55} & c_{56} \\ c_{56} & c_{66} \end{pmatrix} \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial u}{\partial y} \end{pmatrix} \tag{3}$$

The equation of motion for steady state is

$$-\rho\omega^2 u = \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} = c_{55}\frac{\partial^2 u}{\partial z^2} + c_{66}\frac{\partial^2 u}{\partial y^2} + 2c_{56}\frac{\partial^2 u}{\partial y \partial z} \tag{4}$$

where ρ is density and ω is angular frequency.

Mindlin showed that the following form of displacement is a solution of (4). [1] (Notations of some parameters are changed from Mindlin's original paper.)

$$\begin{aligned} u = &\sin\eta y[A\cos\zeta(\beta y - z) + B\sin\zeta(\beta y - z)] \\ &+ \cos\eta y[C\cos\zeta(\beta y - z) + D\sin\zeta(\beta y - z)] \end{aligned} \tag{5}$$

provided

$$\rho\omega^2 = c_{66}\eta^2 + \gamma_{55}\zeta^2 \tag{6}$$

where $\gamma_{55} = c_{55} - \frac{c_{56}^2}{c_{66}}$ and $\beta = \frac{c_{56}}{c_{66}}$ (7)(8)

The first term of (5), (A and B), is an odd mode, whereas the second term, (C and D), is an even mode with respect to the center plane.

Mindlin satisfied traction-free boundary conditions on the upper and the lower major surfaces by

$$\eta h = m \frac{\pi}{2} \tag{9}$$

where h is a half thickness. m is an odd integer for solutions A and B including thickness shear mode and an even integer for solutions C and D including zero, which corresponds to a face shear mode.

Mindlin further satisfied traction-free boundary conditions on side surfaces by the following conditions.

$$\tan \alpha = \beta \tag{10}$$

and

$$\zeta c = n \frac{\pi}{2} \tag{11}$$

where n is an even intger for solutions A and C and an odd integer for solutions B and D. This angle is called Mindlin's angle to honor his discovery of a rare exact solution of vibration of an anisotropic plate.

Substitution of (9) and (10) into (6) yields a resonant frequency, f, of each mode (m, n) for solutions A(odd, even), B(odd, odd), C(even, even) and D(even, odd). Each mode is independent and uncoupled with others. (1,0) and (0, n) correspond to the fundamental thickness twist mode (like AT cut of quartz) and face shear modes (like DT cut of quartz), respectively.

$$\Omega = \frac{f}{f_0} = \sqrt{m^2 + n^2 \frac{\gamma\gamma}{(c/h)^2}} = \sqrt{m^2 + \frac{n^2}{H^2}} \tag{12}$$

where

Ω : normalized resonant frequency (13)

$\gamma\gamma = \gamma_{55} / c_{66}$: normalized gamma (14)

$$f_0 = \frac{1}{2\,(2h)} \sqrt{\frac{c_{66}}{\rho}} \tag{15}$$

: the resonant frequency of (1, 0) mode

$$f_1 = \frac{1}{2\,(2c)} \sqrt{\frac{\gamma_{55}}{\rho}} \tag{16}$$

: the resonant frequency of (0, 1) mode

$$H = \frac{c/h}{\sqrt{\gamma\gamma}} \quad \text{: normalized width/thickness ratio} \tag{17}$$

The frequency spectrum of exact solution can be expressed by a universal normalized form for any crystal in the last equation of (14) and depicted in Fig. 2. A real frequency spectrum as function of width/thickness ratio can be obtained by multiplying normalized gamma to abscissa value. Fig. 3 shows real frequency spectra of interested regions for AT-cut of quartz

III. PIEZOELECTRICALLY INDUCED CHARGE

Mindlin's solution is for pure elastic case omitting piezoelectric effects. Since electromechanical coupling of quartz is small, however, the solution yields a good approximation of distribution of piezolelectrically induced charge on major surfaces, which is given by the following equation.

$$D_2 = e_{25} S_5 + e_{26} S_6 \tag{18}$$

Since there is no variation along the X axis, we divide an electrode into small strip segments parallel to X axis. The area of a unit segment is proportional to its length along the X axis. Shifts of mutual position of each segment have no effects on the excitation.

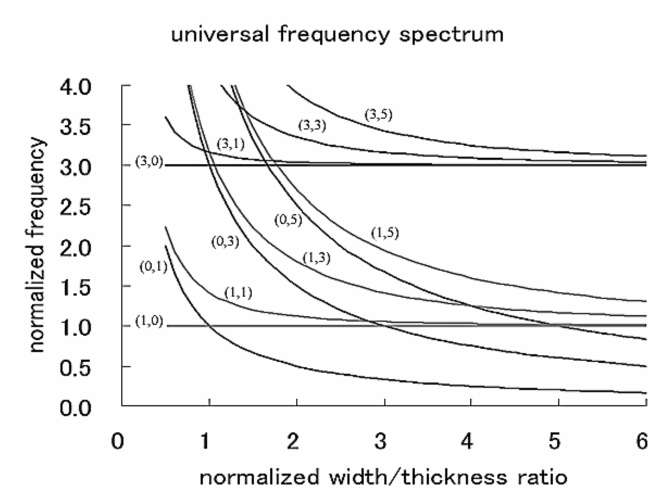

Fig. 2. Universal frequency spectrum of exact solution.

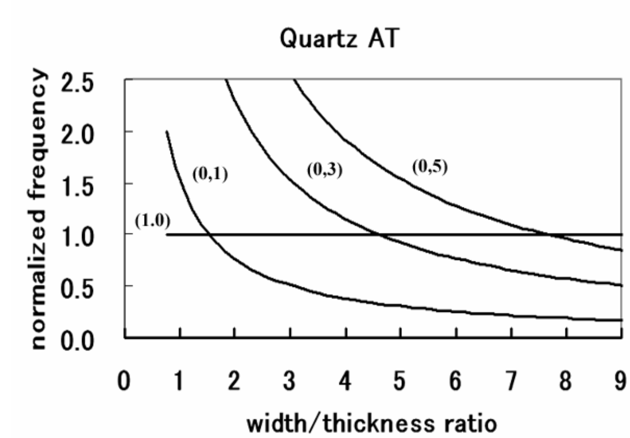

Fig. 3. Frequency spectrum of AT-cut of quartz

We express the length of each segment as a function of z, F(z). F(z) is a projection of area of electrode on Z axis. Then the total charge on an electrode is given by the following integral:

$$DD=\int_{-c}^{c}\int_{-h}^{h} D_2 F(z)\, dydz \tag{19}$$

We consider the case, where F(z) is symmetrical with respect to the center (z = 0). Among the modes given in (5), the following two modes are of present interest.

A: (1,0) mode: the fundamental thickness twist mode.

$$DD=2Ae_{26}\int_{-c}^{c} F(z)dz \tag{20}$$

D: (0,n) mode, n=odd: face shear modes.

$$DD=2D\left(e_{26}-\frac{e_{25}}{\beta}\right)\sin\zeta h\beta\int_{-c}^{c}\cos\left(\frac{n\pi}{2c}z\right)F(z)dz \tag{21}$$

IV. Reduction of Face Shear Modes

If an electrode has the following cosine shape

$$F(z) = x_1 \cos\left(\frac{M\pi}{2c}z\right) \tag{22}$$

where x_1 is arbitrary constant (23)

The integral of all the modes except n= M of face shear modes become null due to the orthogonality of harmonic cosine series. Vormer applied a similar technique to suppress all the overtones of an X-cut resonator at a low frequency. [4]

In the present case of high frequency AT cut, this is rather theoretical interest, because a range of all spurious responses due to face shear modes far exceeds a bandwidth of interest and overlaps with numerous spurious responses due to other modes. Hence it is more practical to reduce not all but a few nearby responses. Adjusting the width of a rectangular electrodes or a combination of several rectangular electrodes, which is easier to fabricate than a cosine electrode, can do this.

V. Experiments

Any one spurious face shear mode can be suppressed by adjusting the width of a conventional rectangular electrode shown in Fig. 4 (a).

$$\left.\begin{array}{l} F(z)=x_2 \quad (|z|\le r) \\ F(z)=0 \quad (r<|z|\le c) \end{array}\right\} \tag{24}$$

where

x_2 : electrode length (arbitrary constant) (25)

$\frac{r}{c}=\frac{2k}{n}$, 2k<n, k is positive integers (26)

: width ratio to eliminate (0,n) face shear mode

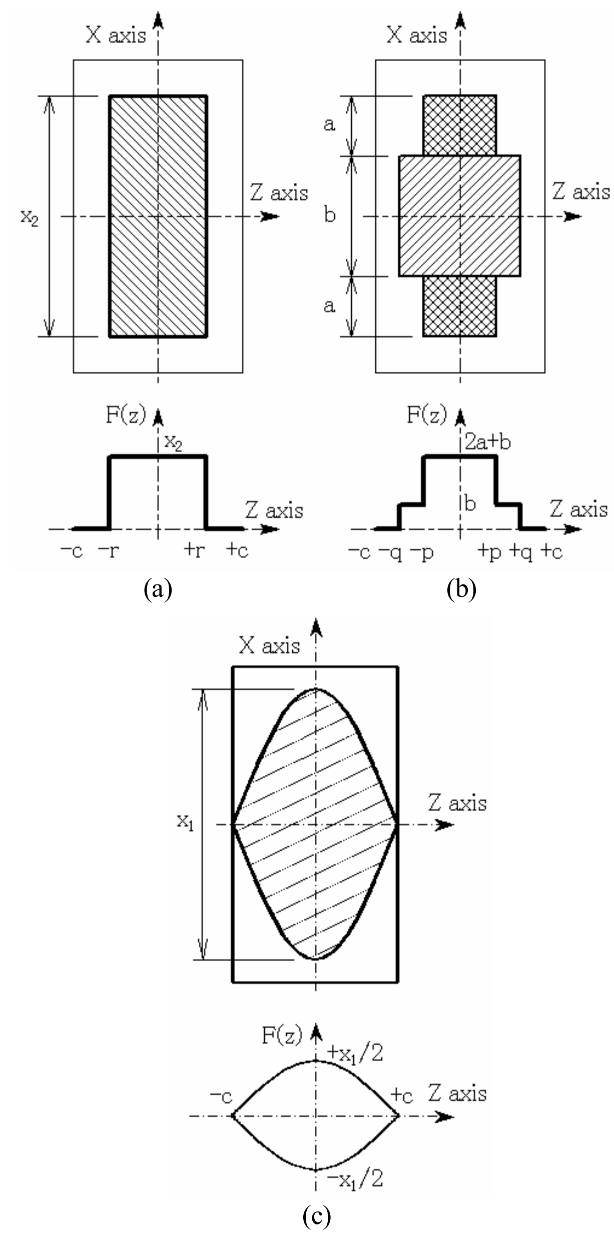

Fig. 4. Dimensions of electrode and its projection, F(z)
(a) rectangular electrode
(b) combination of rectangular electrodes
(c) cosine electrode

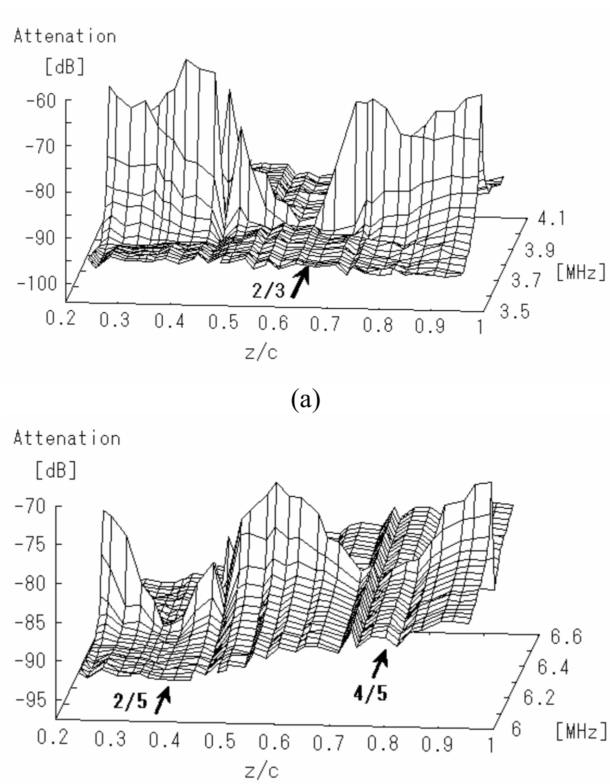

Fig.5 Change of spurious level as function of width ratio
(a) : $n = 3$ and (b) : $n = 5$

z/c : width ratio=electrode width/blank width (27)

Table 1 width ratio to eliminate (0,n) face shear mode

n	width ratio			
3	2/3			
5	2/5	4/5		
7	2/7	4/7	6/7	
9	2/9	4/9	6/9	8/9

Table 1 shows width ratios of electrode to make the integral (21) null for (0,n) face shear mode and therefore suppress its response.

Fig.5 (a) and (b) show the change of experimentally observed spurious levels as functions of the width ratio for n = 3 and 5, respectively. Good suppressions are observed at theoretical values given in Table 1.

Any two spurious face shear modes can be suppressed by a combination of rectangular electrodes shown in Fig. 4 (b).

Two suprious face shear modes $(0,n_1)$ and $(0,n_2)$

$$n_1 \geq 3,\ n_2 \geq 5,\ n_2 > n_1, \quad n_1 \text{ and } n_2 \text{ are an odd integer} \tag{28}$$

$$\left.\begin{aligned} F(z)&=2a+b && (|z|\leq p) \\ F(z)&=b && (p<|z|\leq q) \\ F(z)&=0 && (q<|z|\leq c) \end{aligned}\right\} \tag{29}$$

where

$$\frac{p}{c}=\frac{2k_a}{n_2} \tag{30}$$

: width ratio to eliminate $(0,n_2)$ mode at length a

$$\frac{q}{c}=\frac{2k_b}{n_2} \tag{31}$$

: width ratio to eliminate $(0,n_2)$ mode at length b

$$k_a < k_b,\ 2k_b < n_2, \quad k_a \text{ and } k_b \text{ are positive integers} \tag{32}$$

$$a : b = -\sin\left(n_1\frac{\pi k_b}{n_2}\right) : 2\sin\left(n_1\frac{\pi k_a}{n_2}\right) \tag{33}$$

: length ratio to eliminate $(0,n_1)$ face shear mode

There are many combinations of dimensions of electrodes, which make the integral (21) null for the two modes.

For example, the following parameters are suitable to suppress (0,3) and (0,5) modes together.

The combination $n_1 = 3$, $n_2 = 5$, $k_a = 1$, $k_b = 2$ yield

$a : b = 0.59 : 1.90$.

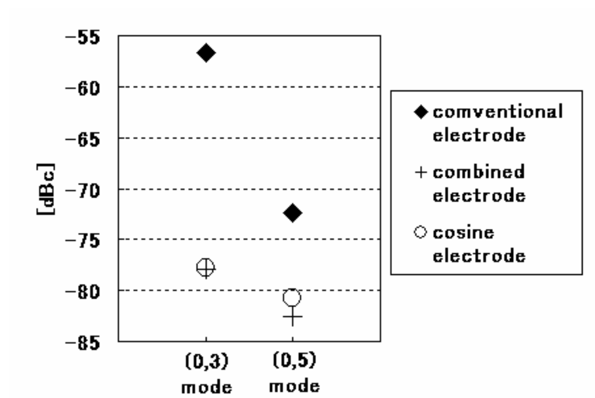

Fig. 6. Comparison of spurious levels of (0,3) and (0,5) modes for a conventional rectangular electrode, a combined electrode and a cosine electrode.

Fig. 6 shows an experimental comparison of spurious levels of (0,3) and (0,5) modes for a conventional rectangular electrode and the above–mentioned combined electrode. A good suppression is observed. Responses of a cosine electrode are also included for a comparison. Its spurious levels are about the same as spurious levels of the combined electrode. Practical merits of a combined electrode are easier fabrication and registration than a cosine electrode.

VI. Reduction of Spurious Modes Caused by Finite Dimension of Length

So far the length along the X axis is assumed as infinitely long. In a real resonator with a finite length along the X axis, there exist spurious responses due to thickness shear modes propagating along the X axis. No exact solution has been available. Approximate solutions [5] [6] and a direct experimental observation of distribution of induced charge [7], however, suggested that the distribution of induced charge along the X axis of these spurious responses also approximately constituted a harmonic cosine series. Hence a similar design of electrodes presented in this paper can be applied to reduce responses of these modes.

It is usual to enhance energy trapping in the center portion by contouring a shape along the X axis and/or mass loading of electrodes. This makes design of electrode more complex. Details will be reported in a subsequent paper.

VII. Conclusion

Miniature AT-cut strip resonators with tilted edges inspired by Mindlin's exact solution of an infinite strip have been widely used. Among four groups of the solution, only face shear modes, mode D, are piezoelectrically excited by electrodes located on major surfaces and cause spurious responses. This paper shows that the distribution of piezoelectrically induced charge along the Z axis constitutes a harmonic cosine series. Hence suitable choice of the width and the shape of electrodes can eliminate piezoelectric excitation of face shear modes. In particular, if the area of electrodes along the X axis is chosen to be proportional to a cosine distribution of a certain mode, responses of all other modes can be eliminated due to the orthogonality of harmonic cosine series. In practice, a rectangular electrode or a combination of several rectangular electrodes, which are easier to fabricate and register, are preferred. Any one face shear mode can be eliminated by adjustment of the width of a rectangular electrode. Plural face shear modes can be eliminated by a suitable choice of dimensional parameters of a combination of several rectangular electrodes. Experiments show a good agreement with the theory.

An extension of a similar design technique is mentioned to the reduction of spurious modes caused by a finite dimension of the length along the X axis.

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References

[1] R. D. Mindlin, "Thickness-twist vibrations of a quartz strip," Proc. 24th Frequency Control Symposium, pp.17-20, 1970.

[2] M. Onoe and M. Okazaki, "Miniature AT-cut strip resonators with tilted edges," Proc. 29th Frequency Control Symposium , pp.42-48, 1975.

[3] M. Onoe, H. Sekimoto and M. Okazaki, "Reduction of spurious responses in a thickness-twist mode resonator made of crystal class 32 by tilting edges," Proc. IEEE UFFC Joint Conference, pp/ 375-380, 2004.

[4] J. J. Vormer, "Crystal plates without overtones, Proc. IRE, vol. 39, pp. 1086-1087, September, 1951.

[5] R.D.Mindlin, "Strong resonances of rectangular AT-cut quartz plate," Proc. 4 th US National Congress of Applied Mechanics, pp. 305-310, 1962.

[6] I. Koga, "Radio-frequency vibrations of rectangular AT-cut quartz plates," Jour. Applied Physics, vol. 34, no. 8, pp.2357-2365, August, 1963.

[7] I. Koga and H. Fukuyo, "Vibration of thin piezoelectric quartz plate (especially on R1-cut rectangular plate)," Jour. Institute of Electronics and Communication Engineers (IECE), vol. 36, no. 2, pp.59-67, Feb., 1963. (In Japanese)